

# Real-time Correlators and Hidden Conformal Symmetry in the Kerr/CFT Correspondence

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**ABSTRACT:** In this paper, we study the real-time correlators in Kerr/CFT, in the low frequency limit of generic non-extremal Kerr(-Newman) black holes. From the low frequency scattering off Kerr-Newman black holes, we show that for the uncharged scalar scattering, there exists hidden conformal symmetry on the solution space. Similar to Kerr case, this suggests that the Kerr-Newman black hole is dual to a two-dimensional CFT with central charges  $c_L = c_R = 12J$  and temperatures  $T_L = \frac{(r_+ + r_-) - Q^2/M}{4\pi a}$ ,  $T_R = \frac{r_+ - r_-}{4\pi a}$ . Using the Minkowski prescription, we compute the real-time correlators of a charged scalar and find perfect match with CFT prediction. We further discuss the low-frequency scattering of photons and gravitons by a Kerr black hole and find that their retarded Green's functions are in good agreement with CFT prediction. Our study shows that hidden conformal symmetry in the solution space is essential to set up and check the Kerr/CFT correspondence.

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## 1. Introduction

The original Kerr/CFT correspondence conjectures that the quantum gravity in the near-horizon extreme Kerr (NHEK) geometry with certain boundary conditions is dual to a (1+1) dimensional chiral conformal field theory (CFT). The correspondence was inspired by the properties of the asymptotic symmetry group of the near horizon geometry [1] of an extreme Kerr black hole where it was found by Guica, Hartman, Song and Strominger (GHSS) [2] that under a certain set of boundary conditions on the asymptotic behaviour of the metric, the  $U(1)_L$  symmetry of the  $SL(2, R)_R \times U(1)_L$  isometry group [5] of the near-horizon geometry get enhanced into a Virasoro algebra. Another copy of Virasoro algebra was later discovered from the analysis of the NHEK boundary condition in [4, 3]. Now it is believed that the extreme Kerr black hole is dual to a CFT with central charges  $c_L = c_R = 12J$ . Support of this conjecture has been found in the perfect match of the macroscopic Bekenstein-Hawking entropy of the black hole with the conformal field theory entropy computed by the Cardy formula. See [6] for some further studies of the Kerr/CFT correspondence as well as generalizations to other spacetime which contain a warped AdS structure.

Further support of the correspondence were found in the studies of the superradiant scattering processes off the extreme Kerr black holes [7]. In this case, the near horizon geometry of a near-extremal Kerr black hole (near-NHEK) is reminiscent of a non-extremal warped black hole. Correspondingly, the right-moving sector of dual CFT is excited [4]. In the near-horizon limit, the modes of interest are the ones near the super-radiant bound. It was shown in [7] that the bulk scattering results were in precise agreement with the CFT description whose form is completely fixed by conformal invariance. Similar discussion has

been generalized to charged Kerr-Newman [8], multi-charged Kerr [9] and higher dimensional near-extremal Kerr black holes. In all these cases, perfect agreements with the dual CFT descriptions have been found.

In [12], it was shown that the real-time correlators of various perturbations in near-extremal Kerr(-Newman) black hole could be computed directly from the bulk, following the Minkowski prescription proposed in AdS/CFT [10] and successfully used in the warped AdS/CFT correspondence[11]. It allowed us to perform a test directly on the CFT correlators and the real-time correlators as obtained by holography. The results are in perfect agreement with the CFT predictions. The similar prescription has been applied to calculate the three-point functions in the Kerr/CFT correspondence[13].

It has been expected that the Kerr/CFT correspondence should be true for general  $J$  and  $M$ , which could be far away from extreme limit. However, for a generic non-extremal Kerr black holes, the NHEK geometry disappears and it is not clear how to associate a CFT from the near horizon geometry. Very recently, in a remarkable paper [14] the authors argued that the existence of conformal invariance in a near horizon geometry is not a necessary condition, instead the existence of a local conformal invariance in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description. The hidden  $SL(2, R) \times SL(2, R)$  conformal symmetry acting on the solution space of the wave function is only a local symmetry, and is broken by the periodic identification in the configuration space. The spontaneous breaking of the conformal symmetry is of the form produced by finite temperatures  $(T_L, T_R)$  in the dual 2D CFT. It was suggested that a generic non-extremal Kerr black hole is dual to a 2D CFT with central charges  $c_L = c_R = 12J$  and temperatures  $T_L = M^2/2\pi J$  and  $T_R = \sqrt{M^4 - J^2}/2\pi J$ . Both the microscopic entropy counting and the low frequency scalar scattering amplitude in the near region support the picture. The other related studies could be found in [15, 16, 17].

In this paper, we would like to study the physical implication of the hidden conformal symmetry on the real-time correlators. Firstly we investigate the low-frequency scattering of the scalar by a Kerr-Newman black hole. We find that for a uncharged massless scalar scattering off the Kerr-Newman black hole, there is a hidden  $SL(2, R) \times SL(2, R)$  conformal symmetry acting on the solution space of the radial wave function. In this case, the dual CFT is of the central charges  $c_L = c_R = 12J$ , but with the temperatures

$$T_L = \frac{r_+ + r_- - Q^2/M}{4\pi a}, \quad T_R = \frac{r_+ - r_-}{4\pi a}. \quad (1.1)$$

We recover the macroscopic Bekenstein-Hawking entropy from the Cardy formula. We discuss the low-frequency massless scalar scattering off the black hole in the near region  $r \ll 1/\omega$ . We use the Minkowski prescription of the AdS/CFT correspondence to calculate the real-time correlators for the charged scalar and find perfect match with the CFT prediction. Then we turn to the low-frequency scattering of vector and gravitational perturbations off a Kerr black hole. Using the prescription proposed in [12], we compute the real-time correlators for these perturbations and once again find perfect agreement with the CFT prediction.

In the next section, we study the low frequency scalar scattering off the Kerr-Newman black hole. In the near region, the wave function takes a form of hypergeometric function, suggesting a underlying conformal invariance. In section 3, we show the hidden  $SL(2, R) \times SL(2, R)$  symmetry in the solution space of the massless uncharged scalar wave function. In section 4, we give the CFT description of the black hole entropy and finite-temperature absorption section of charged scalar, which are in perfect match with the bulk results. In section 5, we discuss the low-frequency scattering of vector and gravitational perturbations off the Kerr black hole. We compute the real-time correlators from the Minkowski prescription, and find the agreement with the CFT prediction. We end with some discussions in section 6.

**Note added:** when we are finishing this project, we notice that there appears a paper [17], which overlaps with our results in section 2 and 3.

## 2. Scalar scattering of Kerr-Newman black hole

For a Kerr-Newman black hole with mass  $M$ , angular momentum  $J = aM$  and electric charge  $Q$ , its metric takes the following form

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a \sin^2 \theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{1}{\rho^2} \sin^2 \theta (adt - (r^2 + a^2)d\phi)^2, \quad (2.1)$$

where

$$\begin{aligned} \Delta &= (r^2 + a^2) - 2Mr + Q^2, \\ \rho^2 &= r^2 + a^2 \cos^2 \theta. \end{aligned} \quad (2.2)$$

The gauge field is

$$A = -\frac{Qr}{\rho^2}(dt - a \sin^2 \theta d\phi). \quad (2.3)$$

There are two horizons located at

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2}. \quad (2.4)$$

And the Hawking temperature, entropy, angular velocity of the horizon and the electric potential are respectively

$$\begin{aligned} T_H &= \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)}, \\ S &= \pi(r_+^2 + a^2), \\ \Omega_H &= \frac{a}{r_+^2 + a^2}, \\ \Phi &= \frac{Qr_+}{r_+^2 + a^2}. \end{aligned} \quad (2.5)$$

Let us consider a complex scalar field with mass  $\mu$  and charge  $e$  scattering off the Kerr-Newman black hole. The Klein-Gordon equation is

$$(\nabla_\mu + ieA_\mu)(\nabla^\mu + ieA^\mu)\Phi - \mu^2\Phi = 0. \quad (2.6)$$

With the ansatz

$$\Phi = e^{-i\omega t + im\phi} \mathcal{R}(r) \mathcal{S}(\theta), \quad (2.7)$$

where  $\omega$  and  $m$  are the quantum numbers, the wave equation could be decomposed into the angular part and the radial part. The angular part is of the form

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d}{d\theta} \mathcal{S} \right) + \left( \Lambda_{lm} - a^2(\omega^2 - \mu^2) \sin^2 \theta - \frac{m^2}{\sin^2 \theta} \right) \mathcal{S} = 0. \quad (2.8)$$

Here  $\Lambda_{lm}$  is the separation constant. It is restricted by the regularity boundary condition at  $\theta = 0, \pi$  and can be computed numerically. The radial part of the wave function is of the form

$$\partial_r(\Delta \partial_r \mathcal{R}) + V_R \mathcal{R} = 0 \quad (2.9)$$

with

$$V_R = -\Lambda_{lm} + 2am\omega + \frac{H^2}{\Delta} - \mu^2(r^2 + a^2), \quad (2.10)$$

$$H = \omega(r^2 + a^2) - eQr - am. \quad (2.11)$$

As we are interested in the low frequency limit,

$$\omega M \ll 1, \quad (2.12)$$

the  $\omega^2$  term in the angular equation could be neglected. Note that the low frequency limit (2.12) is very different from the case studied in [7, 8], where only the frequencies near the superradiant bound were studied. To simplify our discussion, we focus on the massless scalar. Then the angular equation is just the Laplacian on the 2-sphere with the separation constants taking values

$$\Lambda_{lm} = l(l+1). \quad (2.13)$$

In the near region,  $r\omega \ll 1$ , the radial equation could be simplified even more

$$\partial_r \Delta \partial_r \mathcal{R}(r) + \frac{(ma - \omega(2Mr_+ - Q^2) + eQr_+)^2}{(r - r_+)(r_+ - r_-)} \mathcal{R}(r) \quad (2.14)$$

$$- \frac{(ma - \omega(2Mr_- - Q^2) + eQr_-)^2}{(r_+ - r_-)(r - r_-)} \mathcal{R}(r) = (l(l+1) - e^2 Q^2) \mathcal{R}(r) \quad (2.15)$$

Introduce

$$z = \frac{r - r_+}{r - r_-}, \quad (2.16)$$

we can rewrite the equation (2.15) as

$$(1 - z)z \partial_z^2 \mathcal{R}(z) + (1 - z) \partial_z \mathcal{R}(z) + \left( \frac{A_1}{z} + A_2 + \frac{A_3}{1 - z} \right) \mathcal{R}(z) = 0, \quad (2.17)$$

where

$$\begin{aligned} A_1 &= \frac{(ma - \omega(2Mr_+ - Q^2) + eQr_+)^2}{(r_+ - r_-)^2}, \\ A_2 &= - \frac{(ma - \omega(2Mr_- - Q^2) + eQr_-)^2}{(r_+ - r_-)^2}, \\ A_3 &= - (l(l+1) - e^2 Q^2) \end{aligned} \quad (2.18)$$

The equation (2.17) has the solution

$$\mathcal{R}(z) = z^\alpha (1-z)^\beta F(a, b, c; z) \quad (2.19)$$

with

$$\alpha = -i\sqrt{A_1}, \quad \beta = \frac{1}{2}(1 - \sqrt{1 - 4A_3}), \quad (2.20)$$

and

$$c = 1 + 2\alpha, \quad (2.21)$$

$$a = \alpha + \beta + i\sqrt{-A_2}, \quad (2.22)$$

$$b = \alpha + \beta - i\sqrt{-A_2}. \quad (2.23)$$

If  $\beta$  is real, then in outer boundary of the matching region

$$M \ll r \ll \frac{1}{\omega}, \quad (2.24)$$

the solution behaves asymptotically as

$$\mathcal{R}(r) \simeq Ar^{h-1} + Br^{-h} \quad (2.25)$$

where  $h$  is the conformal weight of the scalar field

$$h = 1 - \beta = \frac{1}{2}(1 + \sqrt{(2l+1)^2 - 4e^2Q^2}), \quad (2.26)$$

and

$$A = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad B = \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \quad (2.27)$$

The absorption cross section could be read out

$$P_{abs} \sim |A|^{-2} \sim \sinh \left( 2 \frac{\omega(2Mr_+ - Q^2) - ma - eQr_+}{r_+ - r_-} \right) |\Gamma(h - i(2M\omega - eQ))|^2 \\ \left| \Gamma \left( h - i2 \frac{\omega(2M^2 - Q^2) - ma - eQM}{r_+ - r_-} \right) \right|^2. \quad (2.28)$$

### 3. Hidden conformal symmetry

In this section, we will show that for the massless uncharged particle, there exists a hidden  $SL(2, R)_L \times SL(2, R)_R$  conformal symmetry acting on the solution space. Furthermore, from the spontaneous breaking of this hidden symmetry by periodic identification of  $\phi$ , we can read out the left and right temperatures of the dual conformal field theory.

From the coordinates

$$\omega^+ = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_R \phi + 2n_R t}, \\ \omega^- = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_L \phi + 2n_L t}, \\ y = \sqrt{\frac{r_+ - r_-}{r - r_-}} e^{\pi(T_L + T_R)\phi + (n_L + n_R)t},$$

we can locally define the vector fields

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right) \\ H_{-1} &= i(\omega^{+2}\partial_+ + \omega^+y\partial_y - y^2\partial_-) \end{aligned} \quad (3.1)$$

and

$$\begin{aligned} \tilde{H}_1 &= i\partial_- \\ \tilde{H}_0 &= i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right) \\ \tilde{H}_{-1} &= i(\omega^{-2}\partial_- + \omega^-y\partial_y - y^2\partial_+) \end{aligned} \quad (3.2)$$

These vector fields obey the  $SL(2, R)$  Lie algebra

$$[H_0, H_{\pm 1}] = \mp iH_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0, \quad (3.3)$$

and similarly for  $(\tilde{H}_0, \tilde{H}_{\pm 1})$ . The quadratic Casimir is

$$\begin{aligned} \mathcal{H}^2 = \tilde{\mathcal{H}}^2 &= -H_0^2 + \frac{1}{2}(H_1H_{-1} + H_{-1}H_1) \\ &= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-. \end{aligned} \quad (3.4)$$

In terms of  $(t, r, \phi)$  coordinates, the Casimir becomes

$$\begin{aligned} \mathcal{H}^2 &= (r - r_+)(r - r_-)\frac{\partial^2}{\partial r^2} + (2r - r_+ - r_-)\frac{\partial}{\partial r} \\ &\quad + \frac{r_+ - r_-}{r - r_-} \left( \frac{n_L - n_R}{4\pi A} \partial_\phi - \frac{T_L - T_R}{4A} \partial_t \right)^2 - \frac{r_+ - r_-}{r - r_+} \left( \frac{n_L + n_R}{4\pi A} \partial_\phi - \frac{T_L + T_R}{4A} \partial_t \right)^2 \end{aligned} \quad (3.5)$$

where  $A = n_L T_R - n_R T_L$ . We find that with the following identification

$$\begin{aligned} n_R &= 0, \quad T_R = \frac{r_+ - r_-}{4\pi a} \\ n_L &= -\frac{1}{4M}, \quad T_L = \frac{(r_+ + r_-) - Q^2/M}{4\pi a}, \end{aligned} \quad (3.6)$$

the radial equation (2.15) of neutral scalar, that is  $e = 0$ , is the same as

$$\tilde{\mathcal{H}}^2 \mathcal{R}(r) = \mathcal{H}^2 \mathcal{R}(r) = l(l+1) \mathcal{R}(r). \quad (3.7)$$

In other words, the scalar Laplacian is just the  $SL(2, R)$  Casimir.

As pointed out in [14], the vector fields are not globally defined. In fact, the  $SL(2, R) \times SL(2, R)$  symmetry is spontaneously broken down to  $U(1)_L \times U(1)_R$  subgroup by the periodic identification

$$\phi \sim \phi + 2\pi. \quad (3.8)$$

Note that the temperature identification in (3.6) reflects the nature of the underlying geometry. In the  $Q \rightarrow 0$  limit, it reduces to the one in the Kerr case. It should be universal to all kinds of perturbations, including the charged scalar, even though it was obtained from the radial equation of a massless neutral scalar.

#### 4. Microscopic description

The Kerr/CFT correspondence suggests that the Kerr-Newman black hole is dual to a CFT with central charges

$$c_L = c_R = 12J \quad (4.1)$$

at finite temperature  $(T_L, T_R)$  given in (3.6). This should be true for every value of angular momentum and charge.

As a first check of this conjecture in the Kerr-Newman case, we show that the entropy of the black hole could be recovered from dual CFT. The Cardy formula gives the microscopic entropy

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R). \quad (4.2)$$

From the central charges (4.1) and the temperatures (3.6), we have

$$S = \pi(r_+^2 + a^2) \quad (4.3)$$

which is in perfect agreement with the macroscopic Bekenstein-Hawking area law for the entropy of the Kerr-Newman black hole.

To determine the conjugate charges, we begin with the first law of thermodynamics

$$T_H \delta S = \delta M - \Omega \delta J - \Phi \delta Q. \quad (4.4)$$

We are looking for the conjugate charges  $\delta E_L$  and  $\delta E_R$  such that

$$\delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \quad (4.5)$$

The solution is

$$\begin{aligned} \delta E_L &= \frac{(2M^2 - Q^2)M}{J} \delta M - \frac{QM^2 \delta Q}{J} + \frac{Q^3 \delta Q}{2J}, \\ \delta E_R &= \frac{(2M^2 - Q^2)M}{J} \delta M - \frac{QM^2 \delta Q}{J} - \delta J, \end{aligned} \quad (4.6)$$

If we identify

$$\begin{aligned} \delta M &= \omega, \quad \delta J = m, \quad \delta Q = e, \\ \omega_L &= \frac{(2M^2 - Q^2)M}{J} \omega, \quad \omega_R = \frac{(2M^2 - Q^2)M}{J} \omega - m, \end{aligned} \quad (4.7)$$

$$q_L = q_R = \delta Q = e, \quad (4.8)$$

$$\mu_L = \frac{QM^2}{J} - \frac{Q^3}{2J}, \quad \mu_R = \frac{QM^2}{J} \quad (4.9)$$

we have

$$\delta E_L = \omega_L - q_L \mu_L, \quad \delta E_R = \omega_R - q_R \mu_R. \quad (4.10)$$

In other words, if the charge  $e$  is nonvanishing, it couples to both the left and right chemical potential. If both  $e$  and  $Q$  are vanishing, the above identifications reduce to the ones in the Kerr case.

With the identification (4.7-4.9), the absorption cross section (2.28) could be rewritten as

$$P_{abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L - q_L \mu_L}{2T_L} + \frac{\omega_R - q_R \mu_R}{2T_R}\right) \times \left| \Gamma\left(h_L + i \frac{\omega_L - q_L \mu_L}{2\pi T_L}\right) \right|^2 \cdot \left| \Gamma\left(h_R + i \frac{\omega_R - q_R \mu_R}{2\pi T_R}\right) \right|^2 \quad (4.11)$$

In a 2D conformal field theory(CFT), one can define the two-point function

$$G(t^+, t^-) = \langle \mathcal{O}_\phi^\dagger(t^+, t^-) \mathcal{O}_\phi(0) \rangle, \quad (4.12)$$

where  $t^+, t^-$  are the left and right moving coordinates of 2D worldsheet and  $\mathcal{O}_\phi$  is the operator corresponding to the field perturbing the black hole. For an operator of dimensions  $(h_L, h_R)$ , charges  $(q_L, q_R)$  at temperatures  $(T_L, T_R)$  and chemical potentials  $(\mu_L, \mu_R)$ , the two-point function is dictated by conformal invariance and takes the form [18]:

$$G(t^+, t^-) \sim (-1)^{h_L+h_R} \left( \frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R} e^{iq_L \mu_L t^+ + iq_R \mu_R t^-}. \quad (4.13)$$

The CFT absorption cross section could be defined with the two-point functions, following Fermi's golden rule:

$$\sigma_{abs} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+} [G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)] \quad (4.14)$$

Then after being changed into momentum space, the absorption cross section is

$$\sigma \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L - q_L \mu_L}{2T_L} + \frac{\omega_R - q_R \mu_R}{2T_R}\right) \left| \Gamma\left(h_L + i \frac{\omega_L - q_L \mu_L}{2\pi T_L}\right) \right|^2 \left| \Gamma\left(h_R + i \frac{\omega_R - q_R \mu_R}{2\pi T_R}\right) \right|^2. \quad (4.15)$$

Obviously the absorption cross section of a charged scalar in Kerr-Newman black hole (4.11) is in perfect match with the prediction of dual CFT.

Actually, in the Kerr/CFT correspondence, we can do slightly better. Besides the absorption cross section, we may compare the retarded correlators in the bulk with the Euclidean correlators in dual CFT.

The Euclidean correlator  $G_E$  is obtained by a Wick rotation  $t^+ \rightarrow i\tau_L$ ,  $t^- \rightarrow i\tau_R$ . At finite temperature the Euclidean time is taken to have period  $2\pi/T_L, 2\pi/T_R$  and the momentum space Euclidean correlator is given by

$$G_E(\omega_{L,E}, \omega_{R,E}) = \int_0^{2\pi/T_L} d\tau_L \int_0^{2\pi/T_R} d\tau_R e^{-i\omega_{L,E}\tau_L - i\omega_{R,E}\tau_R} G_E(\tau_L, \tau_R), \quad (4.16)$$

where the Euclidean frequencies are related to the Minkowskian ones by

$$\omega_{L,E} = i\omega_L, \quad \omega_{R,E} = i\omega_R. \quad (4.17)$$

The integral is divergent but can be defined by analytic continuation, one obtains [19]

$$G_E(\omega_{L,E}, \omega_{R,E}) \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{i\frac{\bar{\omega}_{L,E}}{2T_L}} e^{i\frac{\bar{\omega}_{R,E}}{2T_R}} \cdot \Gamma(h_L + \frac{\bar{\omega}_{L,E}}{2\pi T_L}) \Gamma(h_L - \frac{\bar{\omega}_{L,E}}{2\pi T_L}) \Gamma(h_R + \frac{\bar{\omega}_{R,E}}{2\pi T_R}) \Gamma(h_R - \frac{\bar{\omega}_{R,E}}{2\pi T_R}), \quad (4.18)$$

where

$$\bar{\omega}_{L,E} = \omega_{L,E} - iq_L \mu_L, \quad \bar{\omega}_{R,E} = \omega_{R,E} - iq_R \mu_R. \quad (4.19)$$

The retarded correlator  $G_R(\omega_L, \omega_R)$  is analytic on the upper half complex  $\omega_{L,R}$ -plane and its value along the positive imaginary  $\omega_{L,R}$ -axis gives the Euclidean correlator:

$$G_E(\omega_{L,E}, \omega_{R,E}) = G_R(i\omega_{L,E}, i\omega_{R,E}), \quad \omega_{L,E}, \omega_{R,E} > 0. \quad (4.20)$$

This relation holds both for zero and finite temperature. However at finite temperature,  $\omega_{L,E}$  and  $\omega_{R,E}$  take discrete values of the Matsubara frequencies

$$\omega_{L,E} = 2\pi m_L T_L, \quad \omega_{R,E} = 2\pi m_R T_R, \quad (4.21)$$

where  $m_L, m_R$  are integers for bosonic modes and are half integers for fermionic modes.

In [12], it was showed that the real-time correlators could be computed holographically in the bulk. This is feasible because that although the NHEK geometry is more complicated, it is in fact a warped  $\text{AdS}_3$  spacetime with a warping factor being a function of the angular variable. Therefore one can consider the Kerr/CFT correspondence as a generalization of the warped  $\text{AdS}$ /CFT correspondence. However, as the near horizon geometry of generic non-extremal Kerr(-Newman) black holes are far from the warped  $\text{AdS}$  or  $\text{AdS}$  spacetime, it is not certain if we can still apply the prescription to compute the real-time correlators. Nevertheless, the existence of the hidden conformal invariance in the solution space gives us confidence that the Minkowski prescription could be applied even for generic non-extremal Kerr(-Newman) black holes. We now show this is the case.

For a scalar field in a black hole background, the prescription for two-point real-time correlators was first proposed in [10]. It could be simplified as follows. For the scalar wave function satisfying the ingoing boundary condition at the black hole horizon, its asymptotic behavior is

$$\phi \sim A r^{h-1} + B r^{-h}. \quad (4.22)$$

Then taken  $A$  as the source term and  $B$  as the response term, the two-point retarded correlator is just

$$G_R \sim \frac{B}{A} \quad (4.23)$$

For the charged scalar scattering off the Kerr-Newman black hole, its asymptotic behavior looks like (2.25). Therefore its retarded Green's function is just

$$G_R \sim \frac{\Gamma(1-2h)}{\Gamma(2h-1)} \frac{\Gamma\left(h + i\frac{\omega_L - q_L \mu_L}{2\pi T_L}\right) \Gamma\left(h + i\frac{\omega_R - q_R \mu_R}{2\pi T_R}\right)}{\Gamma\left(1-h + i\frac{\omega_L - q_L \mu_L}{2\pi T_L}\right) \Gamma\left(1-h + i\frac{\omega_R - q_R \mu_R}{2\pi T_R}\right)} \quad (4.24)$$

with the identification (4.7-4.9). This is in good match with the CFT prediction (4.20).

## 5. Photons and gravitons in Kerr black hole

In this section, we study the low-frequency scattering of electromagnetic and gravitational perturbation off a Kerr black hole. The metric of the Kerr black hole could be obtained by simply setting  $Q = 0$  in (2.1). To study the perturbations with nonvanishing spin, one has to apply the Newman-Penrose formalism [20]. The problem has been well studied in [21, 22, 23, 24, 25]. It turned out that the equations of motion of the perturbations can be decomposed into two separated equations of motion. The wave function could be decomposed into the form

$$\Psi^s = e^{-i\omega t + im\phi} \mathcal{R}^s(r) \mathcal{S}^s(\theta). \quad (5.1)$$

$\Psi^s$  are related to the electromagnetic field strength and Weyl tensor for spin-1 and spin-2 perturbations. The angular and radial functions satisfy the Teukolsky equations:

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left( \sin\theta \frac{d}{d\theta} \mathcal{S}^s(\theta) \right) + \left( \Lambda_{lm}^s - a^2\omega^2 \sin^2\theta - 2a\omega s \cos\theta - \frac{m^2 + s^2 + 2ms \cos\theta}{\sin^2\theta} \right) \mathcal{S}^s(\theta) = 0, \quad (5.2)$$

$$\Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{d}{dr} \mathcal{R}^s \right) + \left( \frac{H^2 - 2is(r-M)H}{\Delta} + 4is\omega r + 2am\omega + s(s+1) - \Lambda_{lm}^s \right) \mathcal{R}^s = 0, \quad (5.3)$$

where in the Kerr case

$$\begin{aligned} \Delta &= (r^2 + a^2) - 2Mr \\ H &= \omega(r^2 + a^2) - am. \end{aligned} \quad (5.4)$$

Here  $\Lambda_{lm}^s$  is the separation constant, depending on  $l, m, s$  and satisfying

$$\Lambda_{lm}^s(s) = \Lambda_{lm}^s(-s). \quad (5.5)$$

In the low-frequency limit  $\omega \ll 1/M$ , the  $\omega$  dependent terms in the angular equation could be neglected. As a result the separation constant is only a function of  $l, m, s$ . In the near region  $r\omega \ll 1$ , the radial equation reduces to

$$\Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{d}{dr} \mathcal{R}^s \right) + V_s \mathcal{R}^s = 0 \quad (5.6)$$

with

$$V_s = \frac{\sigma_+^2 - 2is(r_+ - M)\sigma_+}{(r - r_+)(r_+ - r_-)} - \frac{\sigma_-^2 - 2is(r_- - M)\sigma_-}{(r - r_-)(r_+ - r_-)} + s(s+1) - \Lambda_{lm}^s, \quad (5.7)$$

where

$$\sigma_+ = 2M\omega r_+ - am, \quad \sigma_- = 2M\omega r_- - am \quad (5.8)$$

The solution satisfying the ingoing boundary condition at the horizon could be once again written in terms of hypergeometric function

$$\mathcal{R} = z^\alpha (1-z)^\beta F(a, b, c; z), \quad (5.9)$$

where  $z = \frac{r-r_+}{r-r_-}$  and

$$\begin{aligned}
\alpha &= -i \frac{\sigma_+}{r_+ - r_-} \\
\beta &= s + \frac{1}{2}(1 + \sqrt{1 + 4\Lambda_{lm}^s}) \\
a &= \frac{1}{2}(1 - \sqrt{1 + 4\Lambda_{lm}^s}) - 2i \frac{2M^2\omega - ma}{r_+ - r_-} \\
b &= s + \frac{1}{2}(1 - \sqrt{1 + 4\Lambda_{lm}^s}) - i(2M\omega) \\
c &= 1 + s - i \frac{2\sigma_+}{r_+ - r_-}.
\end{aligned} \tag{5.10}$$

The asymptotic behavior of the radial wave function is

$$\mathcal{R}^s(r) \sim A^s r^{h-1-s} + B^s r^{-h-s}, \tag{5.11}$$

where

$$\begin{aligned}
h^s &= \frac{1}{2}(1 + \sqrt{1 + 4\Lambda_{lm}^s}) \\
A^s &= \frac{\Gamma(\sqrt{1 + 4\Lambda_{lm}^s})}{\Gamma(s + h^s - i(2M\omega))\Gamma\left(h^s - 2i \frac{2M^2\omega - ma}{r_+ - r_-}\right)} \\
B^s &= \frac{\Gamma(-\sqrt{1 + 4\Lambda_{lm}^s})}{\Gamma(s + 1 - h^s - i(2M\omega))\Gamma\left(1 - h^s - 2i \frac{2M^2\omega - ma}{r_+ - r_-}\right)}
\end{aligned}$$

One may calculate the absorption cross sections following the way in [8] and compare the results with the CFT prediction. It turns out to be in perfect match. We will not present the details here. Instead, we give an alternative derivation from the retarded Green's functions.

For the vector and gravitational perturbations, the prescription has been proposed in [12]. If the radial wave function of the perturbation with the spin  $s$  satisfying the ingoing boundary condition at the black hole horizon has asymptotic behavior as

$$\mathcal{R}^s(r) \sim A^s r^{h-1-s} + B^s r^{-h-s}, \tag{5.12}$$

then the retarded Green's function could be

$$G_R^s \sim \frac{B^{-s}}{A^s}. \tag{5.13}$$

In our case, this leads to

$$G_R^s \sim \frac{\Gamma(1 - 2h^s)}{\Gamma(2h^s - 1)} \frac{\Gamma\left(-s + h^s - i \frac{\omega_L}{2\pi T_L}\right) \Gamma\left(h^s - i \frac{\omega_R}{2\pi T_R}\right)}{\Gamma\left(s + 1 - h^s - i \frac{\omega_L}{2\pi T_L}\right) \Gamma\left(1 - h^s - i \frac{\omega_R}{2\pi T_R}\right)}, \tag{5.14}$$

where we have used the identification (4.7) with  $Q = 0$ . Note that in Kerr case, the chemical potentials  $\mu_{L,R}$  are absent. With the conformal weights of the fields being identified as

$$h_R^s = h^s, \quad h_L^s = h_R^s - s, \tag{5.15}$$

the above retarded Green's function agrees precisely, up to a normalization factor, with the CFT result (4.18) at the Matsubara frequencies. The cross section can be read directly from the above Green's function

$$\begin{aligned}\sigma^s &\sim \text{Im}(G_R^s) \\ &\sim \frac{1}{(\Gamma(h_R^s - 1))^2} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \times \\ &\quad \left|\Gamma\left(h_L^s + i\frac{\omega_L}{2\pi T_L}\right)\right|^2 \cdot \left|\Gamma\left(h_R^s + i\frac{\omega_R}{2\pi T_R}\right)\right|^2.\end{aligned}\tag{5.16}$$

They agree with the CFT result. The similar result has been obtained in the case of four-dimensional black holes in string theory[26].

## 6. Discussions

In this paper, we showed that there existed a hidden conformal invariance in the low-frequency scattering off the Kerr-Newman black holes. Even though the conformal symmetry is broken by periodic identification in the configuration space, it acts on the solution space and associate a dual CFT description to the black hole. More precisely, the conformal coordinate transformation suggests that the generic 4D Kerr-Newman black hole is dual to a 2D CFT with central charges  $c_L = c_R = 12J$  and temperatures  $T_L = \frac{(r_+ + r_-) - Q^2/M}{4\pi a}$ ,  $T_R = \frac{r_+ - r_-}{4\pi a}$ . The Bekenstein-Hawking entropy could be recovered from the Cardy formula counting the microstate degeneracy in the dual CFT. For the charged scalar scattering, we identified the dual operators with conformal dimension, left and right charges and chemical potentials, which allow us to find perfect match with the CFT prediction. Furthermore, we discussed the retarded two-point correlators of vector and gravitational perturbations in the Kerr black hole spacetime. Using the prescription suggested in [12], we computed the retarded Green's functions and found perfect agreement with the CFT Euclidean correlators, which are restricted by conformal invariance. These agreements support the belief that the conformal symmetry in the solution space could be essential to set up a CFT description.

Note that the real-time correlator is closely related to the absorption cross section. Actually the imaginary part of the retarded Green's function gives the greybody factor. On the other hand, the greybody factor could determine the retarded Green's function uniquely via spectral theorem[8]. Therefore, the real-time correlators and the greybody factors should not be taken as independent check of the Kerr/CFT correspondence. Nevertheless, it is nice to see that one can apply the Minkowski prescription of AdS/CFT to obtain the real-time correlators and find good agreement with the CFT predictions.

The fact that the conformal symmetry in the solution space is sufficient to associate a CFT description have profound implications. It may change our viewpoint that some kind of AdS/CFT correspondence should rely on the geometry of the bulk spacetime. It could happen in some parameter region, the solution space has an enhanced conformal symmetry, suggesting a dual CFT description. It would be interesting to understand this picture in other situations, besides the Kerr/CFT correspondence.

The analysis in this paper and other related studies have focused on the low-frequency scattering limit. As it is well known, the dual CFT description has been set up in the near-extremal limit, in which case the focus of the frequency is near the super-radiant bound. These two descriptions are consistent: the same central charges suggesting the same CFT, consistent temperatures, even though the conformal weights of the operators are slightly different. It is interesting to see if we have the same picture for intermediate frequencies.

Our computation focused on the retarded Green's functions. It could be generalized to the three-point functions straightforwardly, following the recipe in [13].

The radial wave functions in all the cases we discussed are of hypergeometric functions. This fact suggests that there could be hidden conformal symmetry acting on the solution spaces of charged scalar, vector and gravitational perturbations. It would be interesting to work them out.

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